Finite-temperature many-body perturbation theory for electrons

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So many areas of physics rely on Richard Feynman's diagrammatic approach, which is ultimately intuitive. There is a formal mathematical justification based on Freeman Dyson's time-dependent perturbation theory, but it is not a constructive theory in the sense that no one can use it to derive higher-order formulas without first exhaustively drawing diagrams using human intuition. Even in quantum chemistry, where exact limits can be reached both in theory and in practice, fourth-order Green's function theory or first-order finite-temperature perturbation theory, which are approximations, have been unknown for many decades. In my talk, I will present a plain and reliable (if tedious) algebraic method of derivation of virtually any perturbation theory and illustrate its application to finite-temperature perturbation theory for electrons, deriving its grand potential, chemical potential, and internal energy formulas at any arbitrary perturbation orders for the first time. I will use them to stipulate diagrammatic rules—bottom up—and prove the linked-diagram theorem. I will also analyze the root cause of its 62-year-old controversy or confusion known as the Kohn–Luttinger nonconvergence problem and offer a simple, practical solution.

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